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Research article

Branching of Measurement Results for Swapping between Two Nonorthogonal Entangled Coherent States

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Abstract

We perform entanglement swapping on two pairs of nonorthogonal entangled coherent states (NECS) of different intensities, and show that the difference in photon densities causes measurement results to branch out into two different outcomes with distinct fidelities.

Keywords: coherent states; entanglement; photon density; swapping**;** fidelity

Introduction

Quantum teleportation[1-5] is the disembodied transport of a physical system that requires, as a prerequisite, the distribution of entangled quantum states between a sender and a receiver. The sender is required to make a joint measurement on her state and an unknown quantum system, thereby obtaining classical information that she transmits to the receiver. Using this information, the receiver transforms his state to recover the original unknown system. In contrast, entanglement swapping entails the transfer of nonlocal correlations[6,7] between quantum systems. Like teleportation, it exploits quantum entanglement[8] and, therefore, requires nonlocal correlations to exist between the quantum states shared by the sender and the receiver. One particle from each entangled pair issubjected to a Bell state measurement, establishing a correlation between them. As a consquence, the other two particles become entangled as well. Zukowski et al.[9] first proposed this protocol for single photon states, while Pan et al.[10] were the first to carry out the procedure experimentally.

We consider the scheme proposed by Prakash et al.[11] for swapping between two pairs of NECS of equal intensities using a beam splitter, two phase shifters, and photon counting measurements. In this paper, we consider quantum states with different photon densities. While it is a well-established and intuitive fact that increasing the number of photons involved in the swapping scheme will affect the fidelity of the teleported state favourably, our calculations demonstrate that a difference in photon densities of states – as opposed to an identical increase in intensities of states with the same initial photon density – causes the expected measurement results to split up into different outcomes with unique fidelities.

Entanglement Swapping Scheme

In this paper we consider three parties: Eve, Alice, and Bob. Eve takes states of different photon densities in modes 1-4, and prepares two pairs of NECS

$$|\phi\rangle_{12} = N_{12}[|\alpha,\alpha\rangle_{12} - z|-\alpha,-\alpha\rangle_{12}], \tag{1}$$

$$|\chi\rangle_{34} = N_{34}[|2\alpha,\alpha\rangle_{34} - |-2\alpha,-\alpha\rangle_{34}], \tag{2}$$

where subscripts denote the modes these states belong to, and z is a complex number. The corresponding normalization constants are

$$N_{12} = [1+|z|^2 - (z+z^*)x^4]^{-1/2} \text{ and } N_{34} = [2(1-x^4)]^{-1/2}, \text{ where } x = e^{-|\alpha|^2}. \tag{3}$$

The intial state of the composite system will be

$$|\psi\rangle_{1234} = |\phi\rangle_{12}|\chi\rangle_{34}. \tag{4}$$

We hope to replace the state in mode 2 with that in mode 4, so that an entangled pair of modes 1 and 4 is formed. Entanglement swapping requires that, starting with $|\phi\rangle_{12}$ and $|\chi\rangle_{34}$, we construct a situation where we end up with $|\phi\rangle_{14}$ and two classical bits. To achieve this, Eve gives Alice and Bob a state each from both the entangled pairs. In our scheme, Alice receives states 2 and 3 while Bob receives states 1 and 4. Alice mixes her initially independent states to entangle them, makes a measurement on her now entangled pair, and obtains two classical bits that she transmits to Bob over a classical channel. Based on the information he receives, Bob chooses a unitary transformation to apply to his states in modes 1 and 4. We show that he ends up with an entangled state $|T\rangle_{14}$ which, if the swapping is perfect, is an exact replica of $|\phi\rangle_{12}$. Figure 1 shows our scheme of entanglement swapping.

In our scheme Alice uses a phase shifter on her state in mode 3, changing state $|\alpha\rangle_3$ to state $|-i\alpha\rangle_5$. She then mixes states in modes 2 and 5 using a 50:50 beam splitter, transforming states $|\beta\rangle_2$ and $|\gamma\rangle_5$ to entangled pairs[12] $\frac{1}{\sqrt{2}}|\beta+i\gamma\rangle_6$ and $\frac{1}{\sqrt{2}}|\gamma+i\beta\rangle_7$. As a result of this process, the independent states with Bob also become entangled. Alice then uses a phase shifter to change state $|\delta\rangle_7$ to state $|-i\delta\rangle_8$. The final state of the whole system becomes

$$|\psi\rangle_{1468} = \frac{N_{12}N_{34}}{\sqrt{2}}[|\alpha,\alpha\rangle_{14}|3\alpha,-\alpha\rangle_{68} - |\alpha,-\alpha\rangle_{14}|-\alpha,3\alpha\rangle_{68}$$

$$-z|-\alpha,\alpha\rangle_{14}|\alpha,-3\alpha\rangle_{68} + z|-\alpha,-\alpha\rangle_{14}|-3\alpha,\alpha\rangle_{68}]. \quad (5)$$

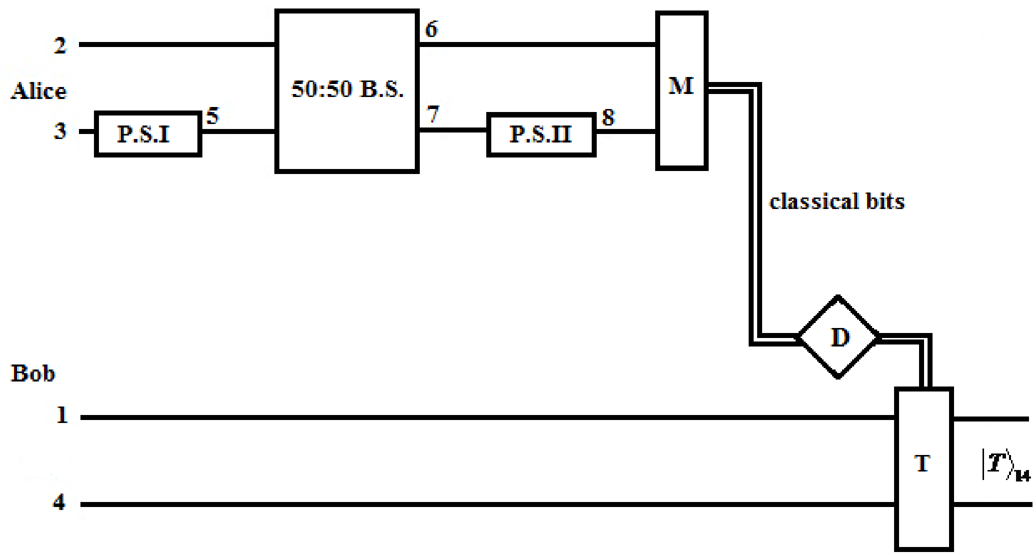

Figure 1: Numerals 1,2,…, 8 refer to modes. The arrow of time goes from left to right. Single lines denote qubits while the double line denotes classical bits. (i) Alice uses phase shifter I to convert state 3 to state 5, (ii) mixes states 2 and 5 using a 50:50 beam splitter, and obtains entangled states 6 and 7, (iii) uses phase shifter II to convert state 7 to state 8, (iv) performs photon counting measuresurement on states 6 and 8, and sends the results to Bob. (v) Based on the information he receives, Bob makes a decision, and (vi) applies a unitary transformation to transfer the original entanglement to his quantum states.

Now Alice and Bob both possess an entangled pair each. This process destroys the quantum channel they initially shared by virtue of being in possession of one half each of two entangled pairs. Alice then makes a measurement on states 6 and 8, which effectively amounts to rewriting them using[11, 13-20] the expression

$$|\pm\alpha\rangle = \sqrt{x}|0\rangle + \frac{(1-x)}{\sqrt{2}}|NZE,\alpha\rangle \pm \sqrt{\frac{1-x^2}{2}}|ODD,\alpha\rangle, \quad (6)$$

where 0, NZE and ODD stand for vacuum, nonzero even and odd number states respectively. Expansion of $|\pm\alpha\rangle$ used in expression (6) has been used by the authors in their earlier work also [11, 13-20]. Note that

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$$|NZE,\alpha\rangle = \frac{|\alpha\rangle + |-\alpha\rangle - 2\sqrt{x}|0\rangle}{\sqrt{2}(1-x)},$$

so that equation 6 becomes

$$|\pm\alpha\rangle = \sqrt{\frac{1+x^2}{2}}|EVEN,\alpha\rangle \pm \sqrt{\frac{1-x^2}{2}}|ODD,\alpha\rangle. \tag{7}$$

For a more precise calculation – or to generate more measurement outcomes – we choose equation 6 over equation 7. Using equation 6 to expand $|\pm\alpha\rangle$ and $|\pm 3\alpha\rangle$, we write the final state of the system as

$$|\psi\rangle_{1468} = \frac{N_{12}N_{34}}{\sqrt{2}}[x^5(|\alpha\rangle_1 - z|-\alpha\rangle_1)(|\alpha\rangle_4 - |-\alpha\rangle_4)|0\rangle_6|0\rangle_8$$

$$+\frac{(1-x)(1-x^9)}{2}(-|\alpha,-\alpha\rangle_{14} - z|-\alpha,\alpha\rangle_{14})|NZE,\alpha\rangle_6|NZE,3\alpha\rangle_8$$

$$+\frac{(1-x)(1-x^9)}{2}(|\alpha,\alpha\rangle_{14} + z|-\alpha,-\alpha\rangle_{14})|NZE,3\alpha\rangle_6|NZE,\alpha\rangle_8$$

$$+\frac{\sqrt{(1-x^2)(1-x^{18})}}{2}(|\alpha,-\alpha\rangle_{14} + z|-\alpha,\alpha\rangle_{14})|ODD,\alpha\rangle_6|ODD,3\alpha\rangle_8$$

$$+\frac{\sqrt{(1-x^2)(1-x^{18})}}{2}(-|\alpha,\alpha\rangle_{14} - z|-\alpha,-\alpha\rangle_{14})|ODD,3\alpha\rangle_6|ODD,\alpha\rangle_8$$

$$+\frac{x^{9/2}(1-x^9)}{\sqrt{2}}(|\alpha,\alpha\rangle_{14} + z|-\alpha,-\alpha\rangle_{14})|0\rangle_6|NZE,\alpha\rangle_8$$

$$+\frac{x^{1/2}(1-x^9)}{\sqrt{2}}(-|\alpha,-\alpha\rangle_{14} - z|-\alpha,\alpha\rangle_{14})|0\rangle_6|NZE,3\alpha\rangle_8$$

$$+\frac{x^{9/2}(1-x)}{\sqrt{2}}(-|\alpha,-\alpha\rangle_{14} - z|-\alpha,\alpha\rangle_{14})|NZE,\alpha\rangle_6|0\rangle_8$$

$$+\frac{x^{1/2}(1-x^9)}{\sqrt{2}}(|\alpha,\alpha\rangle_{14} + z|-\alpha,-\alpha\rangle_{14})|NZE,3\alpha\rangle_6|0\rangle_8$$

$$+x^{9/2}\sqrt{\frac{1-x^{2}}{2}}(-|\alpha,\alpha\rangle_{14}+z|-\alpha,-\alpha\rangle_{14})|0\rangle_{6}|ODD,\alpha\rangle_{8}$$

$$+x^{1/2}\sqrt{\frac{1-x^{18}}{2}}(-|\alpha,-\alpha\rangle_{14}+z|-\alpha,\alpha\rangle_{14})|0\rangle_{6}|ODD,3\alpha\rangle_{8}$$

$$+x^{9/2}\sqrt{\frac{1-x^{2}}{2}}(|\alpha,-\alpha\rangle_{14}-z|-\alpha,\alpha\rangle_{14})|ODD,\alpha\rangle_{6}|0\rangle_{8}$$

$$+x^{1/2}\sqrt{\frac{1-x^{18}}{2}}(|\alpha,\alpha\rangle_{14}-z|-\alpha,\alpha\rangle_{14})|ODD,3\alpha\rangle_{6}|0\rangle_{8}$$

$$+\frac{(1-x)\sqrt{1-x^{18}}}{2}(-|\alpha,-\alpha\rangle_{14}+z|-\alpha,\alpha\rangle_{14})|NZE,\alpha\rangle_{6}|ODD,3\alpha\rangle_{8}$$

$$+\frac{(1-x^{9})\sqrt{1-x^{2}}}{2}(-|\alpha,\alpha\rangle_{14}+z|-\alpha,-\alpha\rangle_{14})|NZE,3\alpha\rangle_{6}|ODD,\alpha\rangle_{8}$$

$$+\frac{(1-x^{9})\sqrt{1-x^{2}}}{2}(|\alpha,-\alpha\rangle_{14}-z|-\alpha,\alpha\rangle_{14}|ODD,\alpha\rangle_{6}|NZE,3\alpha\rangle_{8}$$

$$+\frac{(1-x)\sqrt{1-x^{18}}}{2}(|\alpha,\alpha\rangle_{14}-z|-\alpha,-\alpha\rangle_{14})|ODD,3\alpha\rangle_{6}|NZE,\alpha\rangle_{8}]. \quad (8)$$

Alice then performs photon counting on modes 6 and 8.

Fidelities for Possible Outcomes

We find, for our case, that Alice’s photon counting results in one out of 17 possible outcomes. The photon counting results will be sent to Bob through a classical channel and, depending on the information he receives, Bob will apply a unitary transformation to states 1 and 4 so they become an exact replica of entangled states 1 and 2. We group the outcomes into 6 cases.

Case I: Zero photons in both modes.

Outcome I: Alice measures zero photons in mode 6 and zero photons in mode 8.

We find that Bob possesses the nonentangled state

$$|T'\rangle_{14}\sim(|\alpha\rangle_{1}-z|-\alpha\rangle_{1})(|\alpha\rangle_{4}-|-\alpha\rangle_{4}), \quad (9)$$

that cannot be converted to a replica of $|\phi\rangle_{12}$. The tilde signifies that $|T'\rangle_{14}$ is not normalized. To write equation 9 in terms of even and odd number of coherent states, we use equation 7 and obtain

$$|T'\rangle_{14} = N_I(\sqrt{1-x^4}\,|EVEN,\alpha\rangle_1 + (1-x^2)|ODD,\alpha\rangle_1)|ODD,\alpha\rangle_4, \quad (10)$$

where the normalization constant is $N_I = [(1-x^2)^2(1+z+z^*+|z|^2)]^{-1}$. (11)

Fidelity is defined as

$$F \equiv |\langle\phi|T'\rangle|^2 \quad (12)$$

which, for this case, is

$$F_I = \frac{N_I^2 N_{12}^2}{4}\left|(1-x^4)(1+z^*) + (1-x^2)^2(1-z^*)\right|^2. \quad (13)$$

Case II: Nonzero even number of photons in both modes.

Outcome II: Alice measures nonzero even α photons in mode 6 and nonzero even 3α photons in mode 8. We find that Bob possesses the state

$$|T'\rangle_{14} \sim -|\alpha,-\alpha\rangle_{14} - z|-\alpha,\alpha\rangle_{14}. \quad (14)$$

In terms of even and odd number of coherent states,

$$\begin{aligned}|T'\rangle_{14} = \frac{N_{II}}{2}[&-(1+x^2)(1+z)|EVEN,\alpha\rangle_1|EVEN,\alpha\rangle_4 \\ &+(1-x^2)(1-z)|ODD,\alpha\rangle_1|ODD,\alpha\rangle_4 \\ &+\sqrt{1-x^4}(1+z)|EVEN,\alpha\rangle_1|ODD,\alpha\rangle_4 \\ &-\sqrt{1-x^4}(1-z)|ODD,\alpha\rangle_1|EVEN,\alpha\rangle_4], \end{aligned} \quad (15)$$

where the normalization constant is $N_{II} = [(1+|z|^2)x^4 + (z+z^*)]^{-1/2}$. (16)

Bob applies the unitary transformation

$$U_{II} = -|EVEN,\alpha\rangle_{4\,4}\langle ODD,\alpha| + |ODD,\alpha\rangle_{4\,4}\langle EVEN,\alpha| \quad (17)$$

to obtain the teleported state

$$\left|T\right\rangle_{14} = \frac{N_{II}}{2}[(1+z)\sqrt{1-x^4}\left|EVEN,\alpha\right\rangle_1\left|EVEN,\alpha\right\rangle_4$$

$$-(1-z)\sqrt{1-x^4}\left|ODD,\alpha\right\rangle_1\left|ODD,\alpha\right\rangle_4$$

$$+(1+z)(1+x^2)\left|EVEN,\alpha\right\rangle_1\left|ODD,\alpha\right\rangle_4$$

$$+(1-z)(1-x^2)\left|ODD,\alpha\right\rangle_1\left|EVEN,\alpha\right\rangle_4]. \quad (18)$$

The fidelity is

$$F_{II} = \frac{N_{12}^2 N_{II}^2}{4}(1-x^4)\left|(1+x^2)(1+z)+(1-x^2)(1-z)\right|^2. \quad (19)$$

Outcome III: Alice measures nonzero even 3α photons in mode 6 and nonzero even α photons in mode 8. We find that Bob possesses the state

$$\left|T'\right\rangle_{14} \sim \left|\alpha,\alpha\right\rangle_{14} + z\left|-\alpha,-\alpha\right\rangle_{14} \quad (20)$$

In terms of even and odd number of coherent states,

$$\left|T'\right\rangle_{14} = \frac{N_{III}}{2}[(1+x^2)(1+z)\left|EVEN,\alpha\right\rangle_1\left|EVEN,\alpha\right\rangle_4$$

$$+(1-x^2)(1+z)\left|ODD,\alpha\right\rangle_1\left|ODD,\alpha\right\rangle_4$$

$$+\sqrt{1-x^4}(1-z)\left|EVEN,\alpha\right\rangle_1\left|ODD,\alpha\right\rangle_4$$

$$+\sqrt{1-x^4}(1-z)\left|ODD,\alpha\right\rangle_1\left|EVEN,\alpha\right\rangle_4], \quad (21)$$

where the normalization constant is $N_{III} = [1+|z|^2 + (z+z^*)x^4]^{-1/2}$. (22)

No unitary transformation is required in this case. The fidelity is

$$F_{III} = \frac{N_{12}^2 N_{III}^2}{4}\left|(1+x^4)(1+z-z^*-|z|^2)+(1-x^4)(1-z+z^*-|z|^2)\right|^2. \quad (23)$$

Case III: Odd number of photons in both modes.

Outcome IV: Alice measures odd α photons in mode 6 and odd 3α photons in mode 8. We find that Bob possesses the state

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$$|T'\rangle_{14} \sim |\alpha,-\alpha\rangle_{14} + z|-\alpha,\alpha\rangle_{14}. \tag{24}$$

In terms of even and odd number of coherent states,

$$\begin{aligned}|T'\rangle_{14} = \frac{N_{IV}}{2}[&(1+x^2)(1+z)|EVEN,\alpha\rangle_1|EVEN,\alpha\rangle_4 \\ &-(1-x^2)(1+z)|ODD,\alpha\rangle_1|ODD,\alpha\rangle_4 \\ &-\sqrt{1-x^4}(1-z)|EVEN,\alpha\rangle_1|ODD,\alpha\rangle_4 \\ &+\sqrt{1-x^4}(1-z)|ODD,\alpha\rangle_1|EVEN,\alpha\rangle_4],\end{aligned} \tag{25}$$

where the normalization constant is $N_{IV} = N_{II}$. (26)

Bob applies the unitary transformation

$$U_{II} = |EVEN,\alpha\rangle_{4\,4}\langle ODD,\alpha| - |ODD,\alpha\rangle_{4\,4}\langle EVEN,\alpha| \tag{27}$$

to obtain the teleported state

$$\begin{aligned}|T\rangle_{14} = \frac{N_{IV}}{2}[&\sqrt{1-x^4}(1-z)|EVEN,\alpha\rangle_1|EVEN,\alpha\rangle_4 \\ &+\sqrt{1-x^4}(1-z)|ODD,\alpha\rangle_1|ODD,\alpha\rangle_4 \\ &-(1+x^2)(1+z)|EVEN,\alpha\rangle_1|ODD,\alpha\rangle_4 \\ &+(1-x^2)(1+z)|ODD,\alpha\rangle_1|EVEN,\alpha\rangle_4].\end{aligned} \tag{28}$$

The fidelity is

$$F_{IV} = N_{12}^2 N_{IV}^2 (1-x^4)(1+|z|^2)^2. \tag{29}$$

Outcome V: Alice measures odd 3α photons in mode 6 and odd α photons in mode 8. We find that Bob possesses the state

$$|T'\rangle_{14} \sim -|\alpha,\alpha\rangle_{14} - z|-\alpha,-\alpha\rangle_{14}. \tag{30}$$

In terms of even and odd number of coherent states,

$$|T'\rangle_{14} = -\frac{N_V}{2}[(1+x^2)(1+z)|EVEN,\alpha\rangle_1|EVEN,\alpha\rangle_4$$

$$+(1-x^2)(1+z)|ODD,\alpha\rangle_1|ODD,\alpha\rangle_4$$

$$+\sqrt{1-x^4}(1-z)|EVEN,\alpha\rangle_1|ODD,\alpha\rangle_4$$

$$+\sqrt{1-x^4}(1-z)|ODD,\alpha\rangle_1|EVEN,\alpha\rangle_4], \quad (31)$$

where the normalization constant is $N_V = N_{III}$. (32)

No unitary transformation is required in this case. The fidelity is

$$F_V = F_{III}. \quad (33)$$

Case IV: Zero photons in one mode and nonzero even number of photons in the other.

Outcome VI: Alice measures zero photons in mode 6 and nonzero even α photons in mode 8.

Identical to outcome III.

Outcome VII: Alice measures zero photons in mode 6 and nonzero even 3α photons in mode 8.

Identical to outcome II.

Outcome VIII: Alice measures nonzero even α photons in mode 6 and zero photons in mode 8.

Identical to outcome II.

Outcome IX: Alice measures nonzero even 3α photons in mode 6 and zero photons in mode 8.

Identical to outcome III.

Case V: Zero photons in one mode and odd number of photons in the other.

Outcome X: Alice measures zero photons in mode 6 and odd α photons in mode 8. We find that Bob possesses the state

$$|T'\rangle_{14} \sim -|\alpha,\alpha\rangle_{14} + z|-\alpha,-\alpha\rangle_{14}. \quad (34)$$

In terms of even and odd number of coherent states,

$$|T'\rangle_{14} = \frac{N_X}{2}[(1+x^2)(1-z)|EVEN,\alpha\rangle_1|EVEN,\alpha\rangle_4$$

$$+(1+x^2)(1-z)\left|ODD,\alpha\right\rangle_1\left|ODD,\alpha\right\rangle_4$$

$$+\sqrt{1-x^4}(1+z)\left|EVEN,\alpha\right\rangle_1\left|ODD,\alpha\right\rangle_4$$

$$+\sqrt{1-x^4}(1+z)\left|ODD,\alpha\right\rangle_1\left|EVEN,\alpha\right\rangle_4], \tag{35}$$

where the normalization constant is $N_X = [1+|z|^2 - (z+z^*)x^4]^{-1/2}$. (36)

No unitary transformation is required in this case. The fidelity is

$$F_X = \frac{N_{12}^2 N_X^2}{4}\left|(1+x^4)(1-z-z^*+|z|^2)+(1-x^4)(1+z+z^*+|z|^2)\right|^2. \tag{37}$$

Outcome XI: Alice measures zero photons in mode 6 and odd 3α photons in mode 8. We find that Bob possesses the state

$$\left|T'\right\rangle_{14} \sim -\left|\alpha,-\alpha\right\rangle_{14} + z\left|-\alpha,\alpha\right\rangle_{14}. \tag{38}$$

In terms of even and odd number of coherent states,

$$\left|T'\right\rangle_{14} = \frac{N_{XI}}{2}[-(1+x^2)(1-z)\left|EVEN,\alpha\right\rangle_1\left|EVEN,\alpha\right\rangle_4$$

$$+(1-x^2)(1-z)\left|ODD,\alpha\right\rangle_1\left|ODD,\alpha\right\rangle_4$$

$$+\sqrt{1-x^4}(1+z)\left|EVEN,\alpha\right\rangle_1\left|ODD,\alpha\right\rangle_4$$

$$-\sqrt{1-x^4}(1+z)\left|ODD,\alpha\right\rangle_1\left|EVEN,\alpha\right\rangle_4], \tag{39}$$

where the normalization constant is $N_{XI} = [(1+|z|^2)x^4 - (z+z^*)]^{-1/2}$. (40)

Bob applies the unitary transformation

$$U_{XI} = -\left|EVEN,\alpha\right\rangle_{4\,4}\left\langle ODD,\alpha\right| + \left|ODD,\alpha\right\rangle_{4\,4}\left\langle EVEN,\alpha\right| \tag{41}$$

to obtain the teleported state

$$\left|T\right\rangle_{14} = \frac{N_{XI}}{2}[\sqrt{1-x^4}(1+z)\left|EVEN,\alpha\right\rangle_1\left|EVEN,\alpha\right\rangle_4$$

$$+\sqrt{1-x^4}(1+z)|ODD,\alpha\rangle_1|ODD,\alpha\rangle_4$$

$$+(1+x^2)(1-z)|EVEN,\alpha\rangle_1|ODD,\alpha\rangle_4$$

$$+(1-x^2)(1-z)|ODD,\alpha\rangle_1|EVEN,\alpha\rangle_4]. \tag{42}$$

The fidelity is

$$F_{XI}=\frac{N_{12}^2N_{XI}^2}{4}(1-x^4)(1-|z|^2)^2. \tag{43}$$

Outcome XII: Alice measures odd α photons in mode 6 and zero photons in mode 8. We find that Bob possesses the state

$$|T'\rangle_{14}\sim|\alpha,-\alpha\rangle_{14}-z|-\alpha,\alpha\rangle_{14}. \tag{44}$$

In terms of even and odd number of coherent states,

$$|T'\rangle_{14}=\frac{N_{XII}}{2}[(1+x^2)(1-z)|EVEN,\alpha\rangle_1|EVEN,\alpha\rangle_4$$

$$-(1-x^2)(1-z)|ODD,\alpha\rangle_1|ODD,\alpha\rangle_4$$

$$-\sqrt{1-x^4}(1+z)|EVEN,\alpha\rangle_1|ODD,\alpha\rangle_4$$

$$+\sqrt{1-x^4}(1-z)|ODD,\alpha\rangle_1|EVEN,\alpha\rangle_4], \tag{45}$$

where the normalization constant is $N_{XII}=N_{XI}$. (46)

Bob applies the unitary transformation

$$U_{XI}=|EVEN,\alpha\rangle_{4\,4}\langle ODD,\alpha|-|ODD,\alpha\rangle_{4\,4}\langle EVEN,\alpha| \tag{47}$$

to obtain the teleported state

$$|T\rangle_{14}=\frac{N_{XII}}{2}[\sqrt{1-x^4}(1+z)|EVEN,\alpha\rangle_1|EVEN,\alpha\rangle_4$$

$$+\sqrt{1-x^4}(1-z)|ODD,\alpha\rangle_1|ODD,\alpha\rangle_4$$

$$+(1+x^2)(1-z)|EVEN,\alpha\rangle_1|ODD,\alpha\rangle_4$$

$$+(1-z)(1-x^2)\left|ODD,\alpha\right\rangle_1\left|EVEN,\alpha\right\rangle_4]. \quad (48)$$

The fidelity is

$$F_{XII}=\frac{N_{12}^2 N_{XII}^2}{4}(1-x^4)\left|(1+x^2)(1-z^*-|z|^2)+(1-x^2)(1-z)\right|^2. \quad (49)$$

Outcome XIII: Alice measures odd 3α photons in mode 6 and zero photons in mode 8. We find that Bob possesses the state

$$\left|T'\right\rangle_{14}\sim\left|\alpha,\alpha\right\rangle_{14}-z\left|-\alpha,-\alpha\right\rangle_{14}. \quad (50)$$

In terms of even and odd number of coherent states,

$$\left|T'\right\rangle_{14}=\frac{N_{XIII}}{2}[(1+x^2)(1+z)\left|EVEN,\alpha\right\rangle_1\left|EVEN,\alpha\right\rangle_4$$

$$+(1-x^2)(1+z)\left|ODD,\alpha\right\rangle_1\left|ODD,\alpha\right\rangle_4$$

$$+\sqrt{1-x^4}(1-z)\left|EVEN,\alpha\right\rangle_1\left|ODD,\alpha\right\rangle_4$$

$$+\sqrt{1-x^4}(1-z)\left|ODD,\alpha\right\rangle_1\left|EVEN,\alpha\right\rangle_4], \quad (51)$$

where the normalization constant is $N_{XIII}=N_{III}$. (52)

This state is an exact replica of $\left|\phi\right\rangle_{12}$. No unitary transformation is required in this case. The fidelity is

$$F_{XIII}=F_{III}. \quad (53)$$

Case VI: Nonzero even number of photons in one mode and odd number of photons in the other.

Outcome XIV: Alice measures nonzero even α photons in mode 6 and odd 3α photons in mode 8.

Identical to outcome XI.

Outcome XV: Alice measures nonzero even 3α photons in mode 6 and odd α photons in mode 8.

Identical to outcome X.

Outcome XVI: Alice measures odd α photons in mode 6 and nonzero even 3α photons in mode 8.

Identical to outcome XII.

Outcome XVII: Alice measures odd 3α photons in mode 6 and nonzero even α photons in mode 8.

Identical to outcome XIII.

4. Conclusion

Our results indicate that a difference in the intensities of initial quantum states causes measurement results to split into branches with distinct fidelities. Depending on the initial state of the system, some of these branches may regroup and share the same fidelities. However, the new distribution is not identical to the one we would expect for initial states having the same photon density.

We have isolated the zero state from the even state by dividing the latter into zero and nonzero even. So, we obtain the vacuum state only when the number of photons in both output modes is found to be zero. No swapping or branching of measurement result is observed in this case.

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